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Forum

Probabilities for completely pectinate and symmetric cladograms

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Abstract

Formulas for nonalgorithmically calculating probabilities for completely pectinate and symmetric cladograms are presented.

Completely pectinate and symmetric cladograms containing n taxa are characterized by repeated nested structures; therefore, in addition to recursive (Feller, 1968; Harding, 1971; Heard, 1992; Slowinski and Guyer, 1989; Tajima, 1983) and nonrecursive (Brown, 1994; Stone and Repka, 1998) algorithms, the algebraic prescriptions

$2^{(n-2)}/(n-1)!$(completely pectinate) and

$Z2^{(n-4)}$, if n modulus $2 \neq 0$, or

$Z\Pi(C(2^i - 2, 2^{i-1} - 1)) \wedge 2^{n-i-1}$,

if n modulus $2 = 0$ (completely symmetric),

wherein $Z = (n-2)!/((n-1)!(((n/2)-1)!)^2)$, $C(n,k)$ is the choose function $(n!/((n-k)!k!))$, and the product is calculated over $i = 1, 2, 3, \ldots, \mathrm{Log}_2 n - 1$, may be used to calculate their probabilities as Markovian bifurcation patterns (i.e., probabilities expected on the basis of unbiased bifurcations); these seemingly unwieldy formulas (which can be gleaned by inspecting the Appendix that is presented in Stone and Repka, 1998) yield simple patterned expressions for particular cases, which researchers may use to interpret results that are obtained from cladistic analyses:

n	Completely pectinate	Completely symmetric
4	$2^2/3! = 2/3$	$(2!/((3!)(1!)^2))(C(0,0))^2 = 1/3$
6	$2^4/5! = 2/15$	$(4!/(5!(2!)^2))2^2 = 1/5$
8	$2^6/7! = 4/315$	$(6!/(7!(3!)^2))(C(0,0))^4(C(2,1))^2 = 1/63$
10	$2^8/9! = 2/2835$	$(8!/(9!(4!)^2))2^6 = 1/81$
12	$2^{10}/11! = 4/155925$	$(10!/(11!(5!)^2))2^8 = 4/2475$
14	$2^{12}/13! = 4/6081075$	$(12!/(13!(6!)^2))2^{10} = 4/26325$
16	$2^{14}/15! = 8/638512875$	$(14!/(15!(7!)^2))(C(0,0))^8(C(2,1))^4(C(6,3))^2 = 1/59535$

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