A Crooked Problem Set to Warp Your Mind

During session 22, I described how Euclidean objects can be analysed using equivalent-length, linear segments. Specifically, given equivalent-length, linear segments spanning r units, the dimension D for the Euclidean objects ‘line,’ ‘square,’ and ‘cube’ satisfy the relation $N\ r^D = 1$, wherein N ‘counts’ equivalent components comprising those Euclidean objects (*i.e.*, smaller equivalent lines, squares, or cubes). The relation may be generalised as $N\ r^D = k$, to accommodate other objects or situations.

As I mentioned during session 22, 'fractals' are mathematical object that exhibit nonEuclidean dimensions. Planar fractal curves also satisfy the general relation

$$N = k\ r^{-D},$$

but the exponent D represents a 'fractal dimension,' which may assume any real number. Planar fractal curves 'fill space' to a greater extent than do Euclidean lines (and this space-filling ability increases as D increases); D quantifies complexity, measured as convolution, and ranges between 1.00 and 2.00 for planar fractal curves (D could exceed 2.00 and fill volumes for nonplanar fractal curves – imagine a fractal curve that were ensnared into a tangled ball).

The length L for any curve (Euclidean or fractal) may be determined by approximating it with N equivalent-length, linear segments spanning r units and taking the product

$$L = N\ r.$$

1. Please substitute for N in this equation the expression that is contained in the general relation for N (centred above), to derive a general relation for L as a function involving only the variable r, constant k, and parameter D.

2. Please consider the case D = 1 – a Euclidean line – for this newly derived general relation for L and comment on what k represents in this particular situation.

3. As you are maturing as a Computational Biologist, I reckon that you recognised that the newly derived general relation for L resembles a function that we used earlier in the course as a mathematical model for testing hypotheses about individual growth. Please provide the name for that mathematical model.

4. Please show algebraically how logarithms can be used to perform a linear transformation on the newly derived general relation for L.

5. Please use the data that are copied below to obtain an equation for a best-fit STRAIGHT line, by plotting appropriately TRANSFORMED data (*i.e.*, according to your solution to 4) and including a LINEAR tread line (please show the plot and equation).

r	**L**
10	3014
29	2021
100	2695
205	1101
549	938
988	969

These data were obtained by Lewis F. Richardson, a mathematician who was interested in coastline lengths for countries in Europe long before the word fractal had been coined. The data comprise equivalent-length, linear segments spanning r km and coastline lengths L (in km) for the western coast along Great Britain.

Coastlines may be considered as planar fractal curves. Therefore, coastline lengths L can be calculated by counting minimum 'stepnumbers' N that are required to 'walk' equivalent-length, linear segments spanning r units along coastlines and taking the product N r; you should recall that this product approximates L (as indicated by the equation that you considered in responding to 1). However, as we discussed during sessions 22 (with the images for the island Funafuti) and 24 (with the images depicting elephant teeth), L depends on resolution (*i.e.*, an ant walking along the Funafuti coast would perceive it differently than would a scientist aboard a space shuttle and an ant walking around the perimeter on a leaf that was about to be chewed by an elephant would perceive that perimeter differently than would the elephant). Consequently, N should be considered for a variety of r. Practically, this can be achieved iteratively, by enumerating N for a particular r, calculating L = N r, and decreasing r (*i.e.*, increasing the resolution) with each iteration; measurement resolution is increased (*i.e.*, r is decreased) to the point at which L no longer increases. Calculated in this manner, L approaches numerically the 'actual' coastline or fractal curve length (rather than infinity), as r approaches zero. Whacky!

During session 24, you analysed enamel ridges in photographs depicting elephant teeth (in occlusal-surface-view). These may be considered metaphorically as geographical coastlines. You were asked to measure one specimen.

6. Please provide the Latin binomial for the species that that specimen represents.

7. Please provide a few bits o' information (*e.g.*, one fact) about that species (*e.g.*, from an Internet search or even a book).

8. Please access from the course Internet site (via the 'Administrival' information page) {r, L} data (which were obtained in a manner that is similar to the one that you used during session 24). Please determine the fractal dimension D characterising the enamel ridges for each species.

9. Given that D quantifies complexity, please assess visually whether the morphology for ridges on elephant teeth and D values exhibit any correspondence (*i.e.*, whether

more-complex ridges yield greater D than less complex ridges). Please describe your observations using as few words as you can.

10. 10. Please provide some hypotheses (from your mind or external sources and using as few words as you can) about how D might be related to feeding.

11. Please provide some hypotheses (from your mind or external sources and using as few words as you can) about how D might be related to foraging behaviour.

12. Given the two species that are alive today, please formulate a hypothesis (using your mind or external sources and using as few words as you can) about whether D has increased, decreased, or remained constant during proboscidean evolution.

During session 23, we attempted to derive the Mandelbrot Set via algebraic iteration. During session 24, we investigated the Mandelbrot Set via computer algorithms.

13. Please provide a definition for the Mandelbrot Set and a reference for your source.

14. Please send to me as soon as you can an email message stating “I have read question 10.”