A Chaotic Problem Set for Your Intellectual Growth

We have been considering populations that grow in a discrete manner as prime examples for complex dynamical systems. We chose the logistic difference equation with carrying capacity K = 1

$$N_{t+1} = r_{difference} N_t (1 – N_t)$$

as a model to describe such systems. In this problem set, you will explore this equation from a Computational Biology perspective.

1. A fixed-point is defined as a particular point x_{fixed} for which a function (*e.g.*, f) involving a variable (*e.g.*, x) returns that particular point:

$$f(x_{fixed}) = x_{fixed}.$$

The logistic difference equation may be considered in an analogous manner, by considering N_{t+1} as f and N_t as x; the particular fixed-point would be $N_{t, fixed}$.

Please provide a definition for $N_{t, fixed}$ etymologically (*i.e.*, using only words – as few as you can). You should adopt a biological perspective to achieve this.

2. Please determine the general fixed-point $N_{t, fixed}$ for the logistic difference equation mathematically (*i.e.*, using symbols in an equation). You should derive an equation for N_t (*i.e.*, on the left side) involving only the variable $r_{difference}$ (*i.e.*, on the right side) to achieve this.

3. Predict the range in values that $N_{t, fixed}$ could assume were $r_{difference}$ to reside in the interval between 1 and 3.

4. A fixed-point for a function is ‘hyperbolic’ if the absolute magnitude for the derivative at that particular point is unequal to 1. The derivative for N_{t+1} with respect to N_t is

$$r_{difference} (1 - 2 N_t).$$

Please determine whether the $N_{t, fixed}$ that you predicted in point 3 are hyperbolic.

5. Hyperbolic fixed-points may be classified as ‘attracting’ if the absolute magnitude for the derivative is less than 1 or ‘repelling’ if the absolute magnitude for the derivative is greater than 1. Please classify the $N_{t, fixed}$ that you described in point 3.