FRACTALS & DIMENSIONS

coastline length

FRACTALS

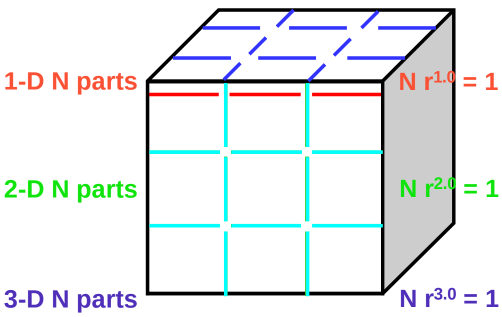
Mandlebrot 1977

wrote *The Fractal Geometry of Nature*
described reality as nonEuclidean
proposed fractal from *fractus* = to break

fractal

shapes are described with reference to dimension
e.g., lightning bolts, dendrites, branchioles, cauliflower, stock-market indices

DIMENSIONS



D

D = - Log[N] / Log[r]
D = - Log[3] / Log[1 / 3] (1-D)
D = - Log[9] / Log[1 / 3] (2-D)
D = - Log[27] / Log[1 / 3] (3-D)

D is a noninteger number for fractals

CANTOR SET

iterative construction
- remove middle-third from the interval [0, 1]
- remove middle-third from remaining intervals
- repeat

properties (at n^{th} iteration)
- 2^n line segments occupying
- $(2 / 3)^n$ total length, so each occupies
- $(1 / 3)^n$ length

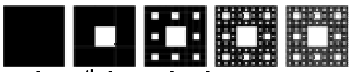
KOCH SNOWFLAKE

iterative construction
- remove middle-third from equilateral triangle edges
- fill gap with another equilateral triangle
- repeat

properties (at n^{th} iteration)
- $3\ 4^n$ sides, each spanning
- 3^{-n} units, so the total perimeter is
- $3\ (4 / 3)^n$ units

SIERPINSKI CARPET

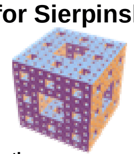
iterative construction
- remove middle-third from a square
- remove middle-third from remaining squares
- repeat



properties (at n^{th} iteration)
- 8^n black boxes, each with side length 3^{-n} units, so the fractional area covered is $(8 / 9)^n$ units

MENGER SPONGE

3-D analogue for Sierpinski Carpet



properties (at n^{th} iteration)
- 20^n filled boxes, each with hole side length 3^{-n} units, so the fractional volume occupied is $(8 / 9)^n$ units