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## BENDING ENERGY MATRIX $L_{p}{ }^{-1}$

$L_{p}^{-1}=E \wedge E^{t}$ $\qquad$
E are 'principal warps'
$\wedge$ is $p \times p$ matrix containing eigenvalues

## WEIGHT MATRIX W

$$
\begin{aligned}
& W=[W x \mid W y] \\
& W=V\left(I_{2} \otimes E \Lambda^{-\alpha / 2}\right) / n^{1 / 2} \\
& V=\left[V_{x} \mid V_{y}\right] \\
& v_{x}=X_{x}-1_{n} \otimes[0 \mid 1] X_{c} \\
& v_{x}=X_{y}-1_{n} \otimes[0 \mid 1] X_{c}
\end{aligned}
$$

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## NORMALISED MATRIX N

$$
\begin{aligned}
& \left.N=W n^{1 / 2}\left(I_{2} \otimes \Lambda^{(1+\alpha / 2}\right) E^{\mathrm{t}}\right) \\
& N=V\left(I_{2} \otimes L_{p}^{-1}\right)
\end{aligned}
$$

## RELATIVE WARP MATRIX R

W = S D R

## THIN-PLATE SPLINE <br> RELATIVE WARP ANALYSIS

Principal Warps are eigenfunctions for bending energy matrix

Partial Warps are obtained by projecting specimens onto principal warps

Relative Warps are principal component axes for space in which each point correspondsto a specimen and axes are inversely weighted principal warps

## PRINCIPAL WARPS

$\mathrm{p} \times \mathrm{p}$ matrix containing eigenvectors with associated eigenvalues that scale inversely with magnitude
pxp-3 if affine components are ignored

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