























 $V_{x} = X_{x} - 1_{n} \otimes [0 \mid 1] X_{c}$

$$V_{x} = X_{y} - 1_{n} \otimes [0 \mid 1] X_{c}$$

WEIGHT MATRIX W

 $\mathsf{W}=\mathsf{V}(\mathsf{I}_2\otimes\mathsf{E}\,\boldsymbol{\Lambda}^{\boldsymbol{-\alpha/2}})\,/\,\mathsf{n}^{1/2}$

W = [Wx | Wy]

 $V = [V_x \mid V_y]$

E are 'principal warps'

 $L_p^{-1} = E \wedge E^t$

 Λ is p x p matrix containing eigenvalues

BENDING ENERGY MATRIX L_p^{-1}





NORMALISED MATRIX N

 $N = W n^{1/2} (I_2 \otimes \Lambda^{(1 + \alpha/2)} E^t)$

 $\mathsf{N} = \mathsf{V}(\mathsf{I}_2 \otimes \mathsf{L}_p^{-1})$

RELATIVE WARP MATRIX R

THIN-PLATE SPLINE RELATIVE WARP ANALYSIS

Principal Warps are eigenfunctions for bending energy matrix

Partial Warps are obtained by projecting specimens onto principal warps

Relative Warps are principal component axes for space in which each point correspondsto a specimen and axes are inversely weighted principal warps



- p x p matrix containing eigenvectors with associated eigenvalues that scale inversely with magnitude
- p x p 3 if affine components are ignored











