A Problem Set for Your Information

1. Please quantify the information that you would acquire by rolling:
    one unbiased die*;
    two dice* that you could distinguish from each other (*i.e.*, one red, one blue);
    two dice* that you would be unable to distinguish from each other (hint: this is tricky; please imagine rolling the two dice and enumerating independently the outcomes in which the upward sides matched and the outcomes in which the upward sides differed).

2. I am the third among 5 children in my immediate family. Suppose that I were to discover that the die* that me and my siblings used to decide who had to 'terrorise the babysitter' when we were children, which, initially, I naively believed to be fair – my siblings mostly became lawyers, was biased so that the probability for yielding the symbol 3 was twice as great as the probability for yielding the other 5 faces, which were equiprobable. Please quantify the information that I would have acquired.

3. Suppose that my sister bore* three children, c1, c2, and c3, and reported to each among 4 absent-minded grandparents a single observation:
    at least one among c1, c2, and c3 is female;
    c1 is female;
    two children are female and one is male
    c2 is male and c1 and c3 are female.
Please quantify the information that my sister conveyed to each grandparent (assume that each grandparent knows that my sister bore three children). Suppose that my sister reported the same observations sequentially and in the same order to a Godfather. Please quantify the information that my sister conveyed with each report. Finally, please compare the total information that was received by the Godfather to the information that was received previously by the fourth grandparent and explain your findings.

4. In textbooks, journals, seminars, and even classrooms, scientists often state that DNA sequences contain information; the information is considered as being encoded in a language comprising 4 different letters representing nucleotide bases. Please quantify the information that is encoded when each single nucleotide base in a sequence is specified.

5. The equation P(j or k) = P(j) + P(k) applies if properties j and k are mutually exclusive. Please derive an equation that applies more generally, even if j and k could occur jointly; the equation should involve only the absolute probabilities P(j) and P(k) and the joint probability P(j and k) (hint: if j and k could occur jointly, then P(j) + P(k) might overestimate P(j or k); please consider by how much).

*All dice that will be considered in this course are 6-sided – in case any among you play adventure games. All babies that will considered in this course are born with P(female) = 0.5 = P(0.5).